Session 5
Thrust

1.0 Definitions

Compressor - The part of an engine which forces the same amount of mass into a smaller volume, increasing the air density.

Combustion - The controlled explosion of a fuel-air mixture.

Disk area - The area described by a propeller as it turns through a full circle.

Turbojet - An engine which has a small intake area (for low drag purposes) that greatly compresses the air, adds fuel and generates rapid air velocities by combustion of the fuel/air mixture.

Turbofan - A engine which is essentially a large fan encased in a shroud mounted on the end of a turbojet shaft.

2.0 Introduction

The previous sessions have developed three of the forces of flight; weight, lift, and drag. Drag was shown as the force acting opposite the flight path of the aircraft, therefore the opposing force, thrust, must act in the same direction as the flight path. However, an engine produces a force which acts toward the rear of the aircraft. Through an application of Newton's third law, this force creates an equal and opposite reaction which "thrusts" the aircraft forward. For this reason, the force produced by the engine is called thrust. This session will describe the origins of thrust and highlight how various engines produce thrust. Thrust may be the most important force because regardless of the type of aircraft being studied (or tested) ALL need some type of thrust to propel them aloft. Even unpowered aircraft such as gliders need a tow plane to provide an external force to pull the aircraft into the air, where it can obtain airflow over the wings to provide the necessary lift to remain airborne. Hang gliders use foot power to initiate movement prior to "leaping" off a cliff. The most common means of developing thrust on powered airplanes comes from propellers or jets. Both of these types employ the same principle of operation involving Newton's second law.

****START VIDEO****

3.0 Principles of Thrust

The explanation of thrust is based entirely on Newton's second law. Recall that force equals the rate of change of momentum:

$$F = \frac{\Delta(mV)}{\Delta t} \tag{5.1}$$

Students will recognize the simplified version of this law that applies when the mass is constant:

$$F = ma$$

for thrust analysis, however, we use equation (5.1) in another form:

$$F = \frac{\Delta(mV)}{\Delta t} = F = \frac{\Delta m}{\Delta t}\Delta V \tag{5.2}$$

$\frac{\Delta m}{\Delta t}$ is known as the mass flow and is sometimes abbreviated as Q. ΔV is simply the total change in velocity of the airflow.

$$F = Q\,\Delta V \tag{5.3}$$

The amount of force, or thrust, generated is dependent upon two primary factors; 1) the amount of mass flow, and 2) the change of the air flow speed.

Each of the primary factors influencing thrust can be varied by different means. If more thrust is required, either the mass flow can be increased or the change in velocity of the air mass as it flows through the propeller can be increased. To create a given amount of thrust, a large amount of mass flow can be accelerated a little or a small amount of mass flow can be accelerated a lot. This concept was demonstrated in the video by the use of paper fans.

3.1 Propeller Aircraft

For a propeller powered aircraft, it can be proven through the use of the kinetic energy theory and the Bernoulli pressure relationship, that the total change of the air flow speed (ΔV) is a function of the aircraft's forward speed and the change in the speed of the air as it immediately passes through the propeller area. This is expressed as:

$$\Delta V = 2\left[\Delta u + \frac{\Delta u^2}{V}\right] \tag{5.4}$$

The $\Delta\upsilon$ is the change in velocity between the air in front of the propeller and the air immediately behind the propeller, as shown in the video.

Figure 5.1 Change in Air Velocity Directly behind Propeller

Caution:

The $\Delta\upsilon$ in this relationship should not be confused with the total change in velocity, ΔV, shown in equation 5.3.

The additional velocity imparted by the propeller was given in the video to be equivalent to a propeller constant, "k" times the engine RPM, giving the relationship:

$$\Delta\upsilon = k\,(\text{RPM}) \qquad (5.5)$$

NOTE:

The propeller constant "k," contains the conversion factor from "revolutions per minute" (RPM) to revolutions per second. "k" is a different constant for each given flight speed and propeller design.

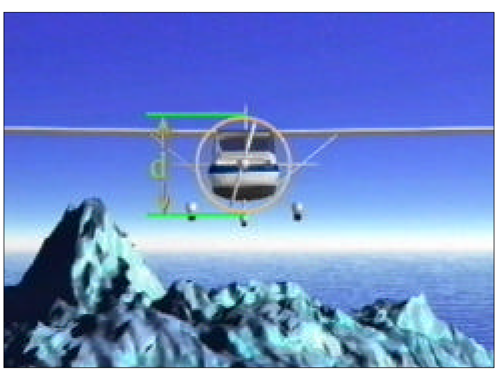

Figure 5.2 Volume of Air

Mass flow (Q) is dependent upon the density speed and area of the air as shown by the relationship:

$$Q = \frac{\{\rho\forall\}}{\Delta t} = \rho A V \qquad (5.6)$$

Recall the area of a circle or disk is:

$$\text{Propeller disk area, } A = \frac{\pi d^2}{4} \qquad (5.7)$$

If we assume the density of the air and the propeller diameter remain essentially constant, then equations 5.3, 5.6 and 5.7 can be combined:

$$F = T = Q\,\Delta V = \rho\left(\frac{\pi d^2}{4}\right)V(\Delta V)$$

Solely for the purpose of helping students relate with accelerations, the video replaced the $V(\Delta V)$ product with a "pseudo-acceleration," a:

$$F = T = \rho\left(\frac{\pi d^2}{4}\right)a \qquad (5.8)$$

If the air's acceleration (a) is replaced with $V(\Delta V)$ [ΔV shown in equation's 5.5 and 5.4] then $a = 2\left[Vk(RPM) + (k\,RPM)^2\right]$. The thrust equation identified in the video combine this and equation 5.8 to get:

$$F = T = \left[\rho\left(\frac{\pi d^2}{4}\right)\right]2\left[Vk(RPM) + (k\,RPM)^2\right] \qquad (5.9)$$

This equation can be quite "messy" therefore, an example may clarify the important points.

****STOP VIDEO****

***Example 1*:**

An aircraft has an engine that can turn 2750 rpm. How much thrust will be generated at 100 mph (147 feet per second) if the propeller diameter is 5 feet and has a "k" value of 0.0044 at an altitude where the density is 0.0022 slugs per cubic foot (which can also be written as $0.0022\frac{lb\,sec^2}{ft^4}$)?

***Solution*:** By directly substituting into equation 5.8 the thrust can be determined.

$$F = T = \left\{\rho\left(\frac{\pi d^2}{4}\right)\right\}2\left\{Vk(RPM) + (k\,RPM)^2\right\} \qquad (5.9)$$

$$T = \left\{0.0022\tfrac{slugs}{ft^3}\left(\frac{\pi 5^2}{4}ft^2\right)\right\}$$
$$2\{147\tfrac{ft}{\sec}(0.0044)(2750\ RPM)$$
$$+(0.0044)(2750\ RPM)^2\}$$

$$T = \left\{0.0022\tfrac{slugs}{ft}\right\}2[147\tfrac{ft}{\sec}(12.1) + 146.41]$$

$$T = \ 166 \text{ lbs}$$

Now to show the effect propeller size has on thrust, consider a forty percent, that is two feet, increase in the propeller diameter.

***Example 2*:**
Using the same aircraft in Example 1, how much thrust can be generated if the propeller diameter is increased to 7 feet?

***Solution*:** Again by direct substitution into equation 5.9, the thrust can be determined.

$$F = T = \left\{\ \rho\left(\tfrac{\pi d^2}{4}\right)\right\}2\{Vk(RPM) + (k\ RPM)^2\} \quad (5.9)$$

$$T = \left\{0.0022\tfrac{slugs}{ft^3}\left(\frac{\pi 7^2}{4}ft^2\right)\right\}$$
$$2\{147\tfrac{ft}{\sec}(0.0044)(2750\ RPM)$$
$$+(0.0044)(2750\ RPM)^2\}$$

$$T = \left\{0.08467\tfrac{slugs}{ft}\right\}2[147\tfrac{ft}{\sec}(12.1) + 146.41]$$

$$T = \ 325 \text{ lbs}$$

This shows that a 40% increase in propeller diameter increased the thrust by 95% as a result of an increase in the mass flow. However, this increase in thrust creates an unbalanced force in the horizontal direction. Recall that in unaccelerated flight, thrust and drag must be equal, according to Newton's third law. The aircraft will therefore accelerate to a new speed where the drag and thrust are again equal. This concept will be covered in further detail in Session 8. For now let's see what happens when the mass flow is kept the same, that is keep the same propeller size, but change the acceleration of the air by changing the RPM.

***Example 3*:**
Using the same aircraft as in example 1, what is the increase in thrust if the propeller RPM is increased to 3000?

***Solution*:** The answer can again be found by direct substitution into equation 5.9.

$$F = T = \left\{\ \rho\left(\tfrac{\pi d^2}{4}\right)\right\}2\{Vk(RPM) + (k\ RPM)^2\} \quad (5.9)$$

$$T = \left\{0.0022\tfrac{slugs}{ft^3}\left(\frac{\pi 5^2}{4}ft^2\right)\right\}$$
$$2\{147\tfrac{ft}{\sec}(0.0044)(3000\ RPM)$$
$$+(0.0044)(3000\ RPM)^2\}$$

$$T = \left\{0.0432\tfrac{slugs}{ft}\right\}2[147\tfrac{ft}{\sec}(13.2) + 1174.241]$$

$$T = \ 182.7 \text{ lbs}$$

This example shows that a 9% increase in RPM (which is really a 9% increase in the change of the flow velocity through the propeller) over that in Example 1 yields a 10% increase in thrust. Highlighted here is the effect of increasing the acceleration of the airflow. Comparing this to Example 2, it would appear that the most effective way to increase thrust is to increase the size of the propeller, which really means increase the mass flow through the propeller. However, as shown in the video, there are practical limits on propeller size. These limits come from the fact that propellers mechanically accelerate the air. This type of acceleration also limits the amount of thrust that can be developed. Jet engines, on the other hand, use an increase in acceleration of the air to create much larger thrust values.

****START VIDEO****

3.2 Jet Engines

In a turbojet engine, the inlet area is small when compared to that of a propeller. As a result, there is a smaller amount of mass entering the engine. Recall previously we assumed the density remained constant. Now in the case of a turbojet, in order to allow for combustion the air density must be increased. This is done by the compressor section of the engine, as shown in the video. As the air progresses toward the rear of the engine, it is forced into the smaller and smaller spaces between the blades of each compressor ring. This compacting of the air results in an increase in the air pressure and density as well as an increase in the air temperature.

Figure 5.3 Jet Engine Compressor

As the air exits the compressor section of the engine, it enters the combustion chamber where fuel is added. This densely packed air/fuel mixture is ignited and the resultant "explosion" accelerates the gases out the rear of the engine at a very high rate of speed. This chemical acceleration of the air (combustion) adds to the thrust produced by the engine. Most jet fighters have a system called afterburners, which adds raw fuel into the hot jet exhaust generating even more thrust through higher accelerations of the air.

So the jet generates large amounts of thrust by chemically accelerating the air as the result of combustion. The fact that the jet compresses the air as much as 40 times (depending upon the number of compressor rings) allows the jet aircraft to fly at higher altitudes where the air is too thin for propeller driven aircraft to fly. These altitudes permit the jet aircraft to fly over most weather systems giving passengers a smoother ride. There is a price to pay for the ability to fly at higher speeds and altitudes. That price comes in the form of higher fuel consumption, or in more everyday terms, lower fuel mileage.

One type of engine is a combination of both the turbojet and a propeller called, appropriately, a turboprop. A turboprop is a small turbojet engine which turns a propeller. The turboprop uses the jet's ability to compress the thin air found at higher altitudes combined with the larger volume of air associated with a propeller to produce modest amounts of thrust at medium altitudes. Although it burns less fuel than a turbojet, it cannot fly as high, nor a fast. Both of these limitations are the result of propeller inefficiencies.

As stated earlier, air density decreases as altitude increases. Since propellers are simply airfoils, they have a tendency to become less effective as the air gets thinner. Additionally, although Examples 2 and 3 proved that increasing the prop size and speed increased thrust, as propellers get bigger and turn faster, the tips begin to reach supersonic speeds. At these tip speeds, shock waves begin to develop and destroy the effectiveness of the prop. It would seem, therefore that the most efficient engine would be a combination of the turbojet and a large, slow turning prop. In recent days, these engines have been developed and are called "high by-pass ratio turbofans."

The engines use a turbojet as a "core" to serve two purposes: 1) produce a portion of the total thrust, and 2) to turn a huge fan attached to the main shaft. The engine can operate at higher altitudes because the jet core can compress the thin air. The thrust produced by the core is supplemented by having a VERY large fan section attached to the main shaft of the core. The fan draws in huge amounts of air and therefore can turn slow enough to prevent the flow at the blade tips from becoming supersonic. The overall result is: 1) the fan mechanically generates a little acceleration to a large amount of air mass, and 2) the jet core compresses thin air and chemically generates large accelerations to relatively small amounts of air. Since the fan is mounted to the same shaft as the core, the by-pass ratio of these engines is determined by dividing the amount of air flowing through the fan blades by the amount of air passing through the engine core. This can be written as:

$$ratio = \frac{(\text{area of fan} - \text{area of core})}{\text{area of core}}$$

Consider the following example using the G.E. 90 engine shown in the video.

***Example 4*:**
If the G.E. 90 engine has a fan diameter of 10.25 feet, and a core diameter of 3.34 feet, what is the bypass ratio of the engine?

Figure 5.4 GE 90 Engine

***Solution*:** The area of the fan is:

$$A = \frac{\pi d^2}{4}$$

$$A = \frac{\pi 10.25^2 ft}{4}$$

$$A = 82.52\ ft^2$$

The area of the core is:

$$A = \frac{\pi 3.34^2 ft}{4}$$

$$A = 8.78\ ft^2$$

Then the bypass ratio is:

$$ratio = \frac{(82.52ft^2 - 8.78ft^2)}{8.78ft^2}$$

$$ratio = \ 8.4 \text{ to } 1$$

This means over eight times as much air moves around the outside of the engine as moves through the engine. Since this air is producing thrust but NOT using fuel directly, the efficiency of the engine is greatly increased.

4.0 Summary

As we have seen, whether an aircraft has a propeller, a turbojet, or a turbofan, all of these produce thrust by accelerating a mass of air to the rear of the aircraft. Let’s finish this session by proving that the movement of this air to the rear creates an unbalanced force pushing the aircraft forward. In the video, a balloon was used to show how when the pressure is equal in all directions there is no net force.

Figure 5.5 Equal Forces in the Balloon

However, when the stem is released, the air escaping from the balloon causes an unbalanced force at the front of the balloon, propelling the balloon forward. The same principle applies to the thrust produced by an aircraft engine. The unbalanced force propels the aircraft forward, creating airflow over the wings which generate lift, causing the aircraft to become airborne. The first step to getting airborne is the takeoff, which it just so happens is the topic of the next session.

5.0 Measures of Performance

1 What is the basic principle of operation behind thrust?

ANSWER

Newton's second law; force equals the rate of change of momentum.

2 What are the two primary factors which determine the amount of thrust which can be generated?

ANSWER

1) The amount of mass flow
2) The change of the air velocity behind the rear of the engine.

3 What are the two ways the thrust can be increased on a propeller driven aircraft?

ANSWER

1) Increase the size of the propeller.
2) Increase the RPM of the propeller.

4 By what means does a propeller accelerate an air mass?

ANSWER

Mechanical

5 By what means does a jet accelerate an air mass?

ANSWER

Chemical or combustion

6 What makes a turbofan engine so efficient?

ANSWER

A smaller turbojet core permits flight at higher altitudes while burning a small amount of fuel. The large fan is connected to the same shaft as the jet and therefore turns while the jet compressors turn. These large fans turn slightly slower while accelerating large amounts of air. This increases the thrust of the engine while not burning any fuel.

6.0 Problems

1 A propeller driven aircraft requires 200 pounds of thrust to fly at 110 miles per hour (161.4 ft/sec). If the engine is capable of turning at 3000 RPM, and the propeller constant (k) is 0.005, how large does the propeller have to be to fly at an altitude where the density is 0.0021 slugs/ft^3?

ANSWER

By rearranging equation 5.9, all the information is given to solve directly for the propeller diameter.

$$F = T = \left\{ \rho\left(\frac{\pi d^2}{4}\right)\right\} 2\{Vk(RPM) + (k\ RPM)^2\} \qquad (5.9)$$

rearranging to solve for "d" yields:

$$d = \sqrt{\frac{2T}{\rho\pi\{Vk(RPM) + (k\ RPM)^2\}}}$$

Now if we let $Vk(RPM) = x$ and $K\ RPM = y$, solving for x and y gives:

$$x = Vk(RPM)$$
$$x = (161.4\tfrac{ft}{\sec})(0.005)(3000RPM)$$
$$x = 2421\tfrac{ft}{\sec}$$

$$y = k\ RPM$$
$$y = (0.005)(3000RPM)$$
$$y = 15\tfrac{ft}{\sec}$$

$$d = \sqrt{\frac{2T}{\rho\pi\{x + y^2\}}}$$

$$d = \sqrt{\frac{2(200lbs)}{\left(0.0020\frac{slugs}{ft^3}\right)(3.14)\{2421\frac{ft}{\sec} + (15\frac{ft}{\sec})^2\}}}$$

$$d = 4.8 \text{ ft}$$

2 What is the size of the "fan" portion of a turbofan engine if the core has a diameter of 2.75 feet and the by-pass ratio is 6.3:1?

ANSWER

Rearranging the relationship

$$ratio = \frac{(\text{area of fan} - \text{area of core})}{\text{area of core}}$$

to solve for the fan area gives:

$$A_{fan} = \{(\text{ratio}) \text{ x } (A_{core})\} + A_{core}$$

Then the area of the core is:

$$A_{core} = \frac{\pi d^2}{4} = \frac{(3.14)(2.75)^2}{4}$$

$$A_{core} = 5.94\ ft^2$$

By substitution into the relationship shown above:

$$A_{fan} = \{(6.3) \text{ x } (5.94 \text{ ft})\} + 5.94 \text{ ft}$$
$$A_{fan} = 43.4\ ft^2$$

Then substituting into the equation for the area of a circle, the fan diameter is determined:

$$A = \frac{\pi d^2}{4}$$

$$d = \frac{4A}{\pi}$$

$$d = \frac{4(43.4ft^2)}{3.14}$$

$$d = 7.4 \text{ ft}$$